CHAPTER THIRTEEN

LINEAR EXPANSION

Coefficient if linear expansion:

- The coefficient of linear expansion or the coefficient of linear expansivity of a substance is defined as the increase in its length per unit length of the substance, for each degree Kelvin rise in temperature.

$$\propto = \frac{Expansion}{Original\ length \times change\ in\ temperature}$$

$$=> \propto = \frac{e}{L_1 \times \triangle T}$$

$$=> \propto = \frac{L_2 - L_1}{L_1(\theta_2 - \theta_1)}$$

where L_1 = initial length.

L_2 = final length.

θ_1 = initial temperature.

θ_2 = final temperature.

The following must also be taken notice of:

(1) e = $\propto L_1(\theta_2 - \theta_1)$
(2) $L_2 = L_1 + \propto L_1(\theta_2 - \theta_1)$
(3) $L_2 = L_1 [1 + \propto (\theta_2 - \theta_1)]$.

Area or superficial expansion:

- This is the increase in the area of a body with temperature change.
- A sheet of metal expands throughout its area, when the temperature is increased over a period of time.

- The area or superficial expansivity, $\beta = \frac{Change\ in\ area}{Original\ area \times temperature\ change.}$

- Also $\beta = \frac{A_2 - A_1}{A_1(\theta_2 - \theta_1)}$, where

(1) A_1 = the original area before the increase in temperature.

If L_1 = the original length before the rise in temperature and b_1 = the original breadth before the rise in temperature then $A_1 = L_1 \times b_1$.

(2) A_2 = Final area after the rise in temperature.

- $A_2 = L_2 \times b_2$, where L_2 = the final length after the rise in temperature, and b_2 = the final breadth after the rise in temperature.

(3) θ_2 = the final temperature after the rise in temperature and θ_1 = the initial temperature before the rise in temperature.

- *The superficial expansivity is* $\beta = 2\propto$, which isapproximately two times the linear expansivity i.e. $\beta = 2\propto$.
- In short for superficiall expansivity $\beta = 2\propto$.
- Also $\beta = \frac{\triangle A}{A_0 \triangle \theta}$, where $\triangle A$ = change in area, A_0 = the original area and $\triangle\theta$ = the change in temperature.

(Q1) The coefficient of linear expansivity of a metal is $2.7 \times 10^{-5}k^{-1}$. If its original area is $600mm^2$, what will be the change in its area if it's temperature changes from 60^0c to 80^0C ?

N/B:

Use $\beta = 2\propto = 2(2.7 \times 10^{-5})$ and $\triangle A$ = change in area.

Soln:

$\propto = 2.7 \times 10^{-5}$

$=> \beta = 2\propto = 2 \times 2.7 \times 10^{-5}$,

$=> \beta = 5.4 \times 10^{-5} = 5.4 \times 10^{-5}k^{-1}$

$A_0 = 600mm^2$ and $\triangle\theta = 80 - 60 = 20^0C$.

But $\beta = \frac{\triangle A}{A_0 \triangle \theta}$, where $\triangle A$ = change in area,

$=> 5.4 \times 10^{-5} = \frac{\triangle A}{600 \times 20}$

$=> 5.4 \times 10^{-5} = \frac{\triangle A}{12000}$,

$=> \triangle A = 5.4 \times 10^{-5} \times 12000$

$= 5.4 \times 10^{-5} \times 12 \times 10^{3}$

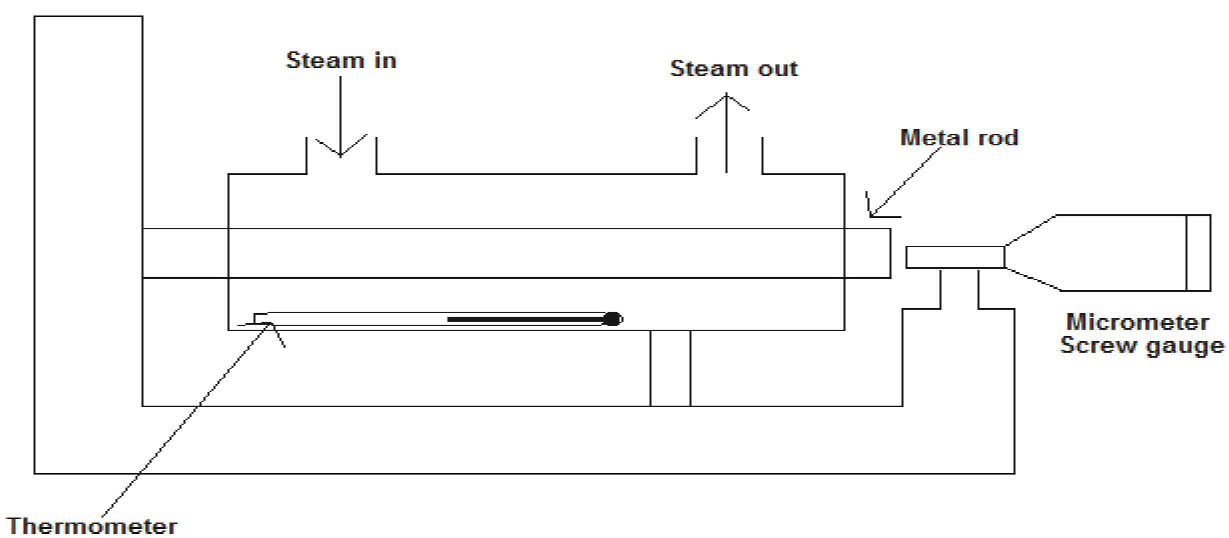
$= 5.4 \times 12 \times 10^{-5} \times 10^{3}$

$= 64.8 \times 10^{-2} = 64.8 \times \frac{1}{100}$

$=> \triangle A = 0.64 mm^2$.

△

Experiment to determine the linear expansivity of a metal rod:



Experiment to determine the linear expansivity of a metal rod:

Procedure:

(1) The metal rod of known length is placed in a chamber, as shown in the given diagram.
(2) Its initial temperature is taken.
(3) Steam is then passed through the chamber until the thermometer reading is steady, and this steady temperature is noted.

(4) The micrometer is then screwed in to make contact with the rod, and its final temperature is needed.

Data:

Initial length of rod = L_0.

Initial temperature of rod = θ_1.

Initial micrometer reading = L_2.

Final micrometer reading = L_3.

Final temperature of rod = θ_2.

Calculation:

Rise in temperature = $\theta_2 - \theta_1$.

Increase in length of rod = $L_3 - L_2$.

Linear expansivity = $\frac{\text{change in length}}{Original\ length \times temperature\ change.}$

$=> \propto = \frac{\text{L3 - L2}}{L_0(\theta_2 - \theta_1)}$.

Precaution: The micrometer should be unscrewed again and the steam flow continued for a few minutes. A final reading of the micrometer is taken in order to be certained whether or not the rod had fully acquired the steam temperature in the first instance.

(Q2) In the determination of the linear expansion of a brass rod, the following results were obtained. Original length of rod brass = 50.2cm.

Initial temperature of the rod = 16.6^0C.

Final temperature of the rod = 99.5^0C.

1^{st} micrometer reading = 3.48mm.

2^{nd} micrometer reading = 4.27mm.

Calculate the coefficient of linear expansion of the rod.

Soln:

Rise in temperature of the rod = 99.5 – 16.6 = 82.9^0C.

Expansion = 4.27 – 3.48 = 0.79mm = $\frac{0.79}{10} = 0.079cm$.

Coefficient of linear expansion = $\frac{Expansion}{Original\ length \times rise\ in\ temperature}$

= $\frac{0.079}{50.2 \times 82.9} = 0.000019/^0$C.

N/B: The original length of the rod is given in centimetre and the expansion is in millimetres. The millimetres must be converted into centimetres by dividing by 10.

<u>Equation for expansion:</u>

- Suppose L_1 = the original length of a rod of the material and $\propto$ = the coefficient of linear expansion for the material, then for a rise θ in temperature, the expansion = $L_1 \propto \theta$.
- The new length, L_2 = the original length + expansion, i.e. $L_2 = L_1 + L_1 \propto \theta$

$=> L_2 = L_1 (1 + \propto \theta)$.

(Q3) Calculate the expansion of a 15m copper pipe, when it is heated from 5^0C to 60^0C, if the linear expansivity of copper = $0.000012k^{-1}$.

Soln:

Original length = L_1 = 15m.

Initial temperature = $\theta_1 = 5^0$C.

Final temperature = $\theta_2 = 60^0$C.

Linear expansivity of copper = $\propto = 0.000012k^{-1}$.

$e = expansion = ?$

But since $\propto = \frac{e}{l_1(\theta_2 - \theta_1)}$

$$=> 0.000012 = \frac{e}{15(60-5)},$$

$$=> 0.000012 = \frac{e}{15 \times 55}$$

$$=> 0.000012 = \frac{e}{825}$$

$$=> \mathrm{e} = 0.000012 \times 825$$

=> e = 9.9×10^{-3} = 0.0099m.

(Q4) The coefficient of linear expansion of iron is 0.000012k^{1}.

(a) Explain the meaning of this statement.
(b) Calculate the superficial expansion of the iron.
(c) Determine the cubical expansion of the iron.

Soln:

(a) It means that the fraction of the original length by which a rod of iron will expand per Kelvin rise in temperature, is 0.000012.
(b) Linear expansivity = $\propto$ = $1.2 \times 10^{-5}k^{-1}$. (i.e. $0.000012k^{-1}$).
But $\beta = 2\propto = 2 \times 1.2 \times 10^{-5}$,
=> $\beta = 2.4 \times 10^{-5}k^{-1}$.

(c) For cubical expansion, Y = $3\propto = 3 \times 1.2 \times 10^{-5} = 3.6 \times 10^{-5}$.

(Q5) A metal rod has a length of 99.4cm at 200^0C. At what temperature will its length be 100cm, if the linear expansivity of the metal is $0.000021k^{-1}$.

N/B: The lengths given in centimetres must be converted into metres by dividing it by 100.

Soln:

Original length $= \mathrm{L_1} = 99.4\mathrm{cm} = \frac{99.4cm}{100} = 0.994m.$

Final length $= \mathrm{L_2} = 100\mathrm{cm} = \frac{100cm}{100} = 1m.$

Initial temperature $= \theta_1 = 200^0\mathrm{C}.$

Final temperature $= \theta_2 = ?$

Linear expansivity = $\propto$ = 2.1 x $10^{-5}k^{-1}$.

But since $\propto = \frac{l_2 - l_1}{l_1(\theta_2 - \theta_1)}$

=> 2.1 x 10^{-5}= $\frac{1-0.994}{0.994(\theta_2-200)}$,

=> $0.000021 = \frac{0.006}{0.994(\theta_2-200)}$

=> $0.000021 \text{ x } 0.994(\theta - 200) = 0.006$,

=> $0.000021(\theta - 200) = \frac{0.006}{0.994}$,

=> 2.1 x $10^{-5}(\theta_2 - 200) = 0.006$,

=> $\theta_2 - 200 = \frac{0.006}{2.1 \times 10^{-5}}$

=> $\theta_2 - 200 = \frac{0.006}{2.1}$ x 10^5

=> $\theta_2 - 200 = \frac{600}{2.1}$

=> $\theta_2 - 200 = 286$

=> θ_2 = 286 + 200 = 486^0C.

(Q6) The length of a wire at a temperature of 30.0^0C is 1.002m. If the temperature of the wire is raised to 105.0^0C and the linear expansivity of the wire is 1.89 x $10^{-5}k^{-1}$, find the increase in the length of the wire.

Soln:

L_1 = 1.002m, θ_2 = 105^0C, θ_1 = 30^0C, $\propto$ = 1.89 x $10^{-5}k^{-1}$,

But since$\propto$= $\frac{l_2 - l_1}{l_1(\theta_2 - \theta_1)}$ => $l_2 - l_1 = \propto l_1 (\theta_2 - \theta_1)$, but since $l_2 - l_1 = e$

=> $e = \propto l_1(\theta_2 - \theta_1)$,

=> e = 1.89 x 10^{-5} x 1.002(105 – 30),

=> e = 1.89 x 1.002 x 10^{-5} x 75,

$\Rightarrow e = 1.89 \times 1.002 \times 75 \times 10^{-5}$,

$\Rightarrow e = 142.034 \times 10^{-5}$,

$\Rightarrow e = 142 \times 10^{-5}$

$\Rightarrow e = 142 \times 10^{-2} \times 10^{-3}$,

$\Rightarrow e = 142 \times \frac{1}{10^2} \times 10^{-3}$

$\Rightarrow e = \frac{142}{100} \times 10^{-3}$

$\Rightarrow e = 1.42 \times 10^{-3}$m.